New strategies for avoiding robot joint limits: Application to visual servoing using a large projection operator

Mohammed Marey, François Chaumette

The 2010 IEEE/RSJ International Conference on
Intelligent Robots and Systems
October 18-22, 2010, Taipei, Taiwan

New strategies for avoiding robot joint limits: Application to visual servoing using a large projection operator

Mohammed Marey & François Chaumette

Abstract— **In this paper, we present a new redundancy-based strategy for avoiding joint limits of a robot arm. This strategy is based on defining three functions for each joint: an activation function; an adaptive gain function; and a tuning function. These functions allow determining automatically the required sign and the suitable magnitude for the avoidance process at each joint. The problem of adding an additional task with the main task and the avoidance process is also considered and solved. As for the redundancy framework, a new large projection operator based on the norm of the usual error is used to enlarge the redundancy domain for applying our proposed avoidance strategy. The experimental results obtained on a 6 dof robot arm in eye-in hand visual servoing show that the new avoidance strategy gives smooth joint avoidance behavior without any tuning step. Using the new projection operator allows a significant improvement of the joint avoidance process, especially in the case of a full rank task function.**

I. INTRODUCTION

Joints limit avoidance is a classical and crucial issue in robot control. The utilization of redundancy has been widely used for solving this problem. The general solution by this technique is obtained as a minimum norm solution together with a homogeneous solution, which is referred to as self-motion [1-7]. The gradient projection method is classical to avoid the joint limits, where the gradient of a performance criterion is projected on to the null space to obtain self-motion [10-11]. We will refer to this method as the classical approach in the following.

Different other strategies have been used to solve the avoidance problem. In [12], a global objective function that realizes a compromise between the main task and secondary tasks is used by exploiting the robot redundant DOFs with respect to the main task. This approach was used to avoid joint limits in a target tracking system. However, important perturbations can be produced by the obtained motions, which are generally not compatible with the regulation to zero of the main task. Also, the global task can fail when the same joints are used for the avoidance and for achieving the main task. In [11], a redundancy-based iterative approach is proposed to avoid the robot joint limits. This method does not affect the main task achievement and ensures the avoidance problem by automatically generating a robot motion compatible with the main task by iteratively solving a system of linear equations to cut any motion on the axis that are in a critical situation. Finally, a nonlinear projection operator has been recently proposed in [9] to improve the performance of avoidance control law. In all the above methods, joint limit avoidance can only be performed if the main task does not constrain all robot DOFs, ie. the main task is not full rank. If it is full rank the classical projection operator is equal to zero and it is impossible to consider any secondary avoidance task. We will see that the methods we propose in this paper leads to significant improvements.

Mohammed Marey and François Chaumette are with INRIA, Centre Rennes-Bretagne Atlantique, IRISA, Campus de Beaulieu, F35042 Rennes-cedex, France. E-mail: `Firstname.Name@irisa.fr`

Mohammed Marey is granted by the Egyptian Government.

Three main issues are presented. The first issue is concerned with the new projection operator proposed in [14] for injecting the avoidance task into the main task, which enhances the degree of the injection into the main task. The second issue is concerned with presenting a new avoidance technique that uses the gradient projection method. This avoidance technique is based on three functions proposed: an activation function that activates the avoidance task and sets its direction; an adaptive gain function that controls the magnitude of the avoidance task; and a tuning function that ensures the smoothness of the injection of the avoidance task into the main task. The third issue is concerned with solving the problem of adding additional secondary tasks to the main task to be performed simultaneously while ensuring the joint limits avoidance. These additional tasks can be used for moving the robot away from the neighborhood of the joint limits, avoiding occlusion, avoiding obstacles, or performing emergency needed motion.

The paper is organized as follows: In Section II, the classical and the new projection operators are recalled. In Section III, the new avoidance strategy is presented and discussed. In Section IV, the problem of adding additional secondary tasks to the avoiding process is considered. Finally, the experimental results are presented in Section V.

II. GRADIENT PROJECTION APPROACHES

Let $\mathbf{e} \in \mathbb{R}^m$ be the main task function where m is the number of its components. The classical approach trying to ensure an exponential decrease of all components of $\mathbf{e}$ leads to the following control scheme [6]:

$$\begin{aligned} \dot{\mathbf{q}} &= \dot{\mathbf{q}}_1 - \mathbf{P_e}\,\mathbf{g} & (1)\\ &= -\lambda \mathbf{J_e}^+\mathbf{e} - (\mathbf{I}_n - \mathbf{J_e}^+\mathbf{J_e})\,\mathbf{g} & (2) \end{aligned}$$

where $\dot{\mathbf{q}}$ is the articular velocity sent as inputs of the low level robot controller, $\dot{\mathbf{q}}_1 = -\lambda \mathbf{J_e}^+\mathbf{e}$ is the articular velocity to perform the main task, $\mathbf{J_e} \in \mathbb{R}^{m\times n}$ is the task Jacobian defined such that $\dot{\mathbf{e}} = \mathbf{J_e}\dot{\mathbf{q}}$, n is the number of robot DOFs, $\mathbf{J_e}^+$ is the Moore-Penrose pseudoinverse of $\mathbf{J_e}$, $\mathbf{g}$ represents the motion induced by the secondary task, and $\mathbf{P_e} = \mathbf{I}_n - \mathbf{J_e}^+\mathbf{J_e}$ is a projection operator on the null space of $\mathbf{J_e}$ so that $\mathbf{g}$ is realized at best under the constraint that it does not perturb the regulation of $\mathbf{e}$ to $\mathbf{0}$. The classical projection operator may be too much constraining: it has only $n-r$ components available (where r is the rank of $\mathbf{J_e}$). Typically when the error $\mathbf{e}$ constrains all the n DOFs of the system, $\mathbf{P_e}$ is equal to $\mathbf{0}$, which prevents considering any secondary task. The new projection operator $\mathbf{P}_{\|\mathbf{e}\|}$ proposed in [14] allows solving this problem when $\mathbf{e} \neq \mathbf{0}$. $\mathbf{P}_{\|\mathbf{e}\|}$ is obtained from the norm of the total error $\|\mathbf{e}\|$ as the projection operator on the null space of $\mathbf{J}_{\|\mathbf{e}\|}$. It is given by:

$$\mathbf{P}_{\|\mathbf{e}\|} = \mathbf{I}_n - \frac{1}{\mathbf{e}^\top \mathbf{J_e} \mathbf{J_e}^\top \mathbf{e}} \;\; \mathbf{J_e}^\top \mathbf{e}\mathbf{e}^\top \mathbf{J_e} \tag{3}$$

This projection operator is always at least of rank $(n-1)$. Hence it can be used even if the main task is full rank. This large projection operator makes it possible to project a secondary task into any main task while ensuring an exponential decrease of its norm. It is well defined as soon as $\mathbf{e} \neq \mathbf{0}$ or $\mathbf{e} \notin \text{Ker}(\mathbf{J_e}^\top)$. That is why a switching strategy from $\mathbf{P}_{\|\mathbf{e}\|}$ to $\mathbf{P_e}$ when $\mathbf{e} \to \mathbf{0}$ has been proposed in [14]. In the following, we will of course use $\mathbf{P}_{\|\mathbf{e}\|}$ instead of $\mathbf{P_e}$ in (1).

III. Joint limits avoidance

To avoid the problem of reaching a joint limit the control law used is given by:

$$\dot{\mathbf{q}} = \dot{\mathbf{q}}_\mathbf{1} + \dot{\mathbf{q}}_\mathbf{2} \tag{4}$$

where $\dot{\mathbf{q}}_\mathbf{2}$ is the articular velocity to perform the secondary task for joint limits avoidance. It is given by:

$$\dot{\mathbf{q}}_2 = \sum_{i=1}^{n} \dot{\mathbf{q}}_\mathbf{2}^i \tag{5}$$

where $\dot{\mathbf{q}}_\mathbf{2}^i$ defines the avoidance task for the i^{th} joint.

A. Classical approach

Let us denote the value of the i^{th} joint j_i by q_i. The lower and upper limits for each joint j_i are denoted $q_i^{\min}$ and $q_i^{\max}$. The configuration $\mathbf{q}$ of the robot is said to be safe with respect to its joint limits if for all joints j_i, $q_i \in [\, q_{\ell_0 i}^{\min}, q_{\ell_0 i}^{\max}\,]$, where

$$q_{\ell_0 i}^{\min} = q_i^{\min} + \rho \Delta q_i \tag{6}$$

$$q_{\ell_0 i}^{\max} = q_i^{\max} - \rho \Delta q_i \tag{7}$$

define the safe domain length of articulation j_i with $\rho \in [0, \frac{1}{2}]$ is a tuning parameter (typically $\rho = 0.1$) and $\Delta q_i = q_i^{\max} - q_i^{\min}$. In the classical approaches [7], [8], [9] a cost function h_s is designed to be minimal at safe configuration and maximal in the vicinity of the joint limits. It is generally defined by:

$$h_s = \frac{\beta}{2} \sum_{i=1}^{n} \frac{{\Delta_i}^2}{\Delta q_i} \tag{8}$$

with

$$\Delta_i = \begin{cases} q_i - q_{\ell_0 i}^{\min}, & \text{if } q_i < q_{\ell_0 i}^{\min} \\ q_i - q_{\ell_0 i}^{\max}, & \text{if } q_{\ell_0 i}^{\max} < q_i \\ 0, & \text{else} \end{cases} \tag{9}$$

where β is a constant to set the amplitude to the control law due to the secondary task. The classical avoidance task is thus given by:

$$\dot{\mathbf{q}}_\mathbf{2}^i = -\mathbf{P_e} \mathbf{g}_i \tag{10}$$

where $\mathbf{g}_i = \frac{\Delta_i}{\Delta q_i}$.

In these classical methods, the tuning of β is extremely difficult. If too large, it results in oscillations. If too small, it can not allow avoiding the joint limits. Also, a suitable value for a given configuration may be either too large or too small for other configurations.

B. New approach

Our new proposed scheme is given by:

$$\dot{\mathbf{q}}_\mathbf{2}^i = -\lambda_{sec_i} \lambda_{\ell_i} \mathbf{P} \mathbf{g}_i^\ell \tag{11}$$

where $\mathbf{P}$ is either the classical projection operator $\mathbf{P_e}$, or the new one $\mathbf{P}_{\|\mathbf{e}\|}$, λ_{sec_i} is an adaptive gain function to control the magnitude of the avoidance task, $\lambda_{\ell_i}(q_i)$ is a tuning function to ensure the smoothness of injecting the avoidance task into the main task, $\mathbf{g}_i^\ell$ is a vector indexing function that controls the activation of the avoidance task and determines its sign.

1) Activation and sign function: If the configuration is not safe with respect to the joint limits, a secondary task for joint limit avoidance is activated by the vector $\mathbf{g}_i^\ell = (\mathbf{g}_i^\ell[1], \mathbf{g}_i^\ell[2], ..., \mathbf{g}_i^\ell[n])^\top$ where each function $\mathbf{g}_i^\ell$ is defined by:

$$\mathbf{g}_i^\ell[i_0] = \begin{cases} -1, & \text{if } q_i < q_{\ell_0 i}^{\min} \text{ and } i = i_0 \\ 1, & \text{if } q_{\ell_0 i}^{\max} < q_i \text{ and } i = i_0 \\ 0, & \text{else} \end{cases} \tag{12}$$

where $i_0 \in \{1, 2, ..., n\}$. If $\mathbf{g}_i^\ell[i_o] = 0$, no avoidance task for the joint j_i is activated. The values 1 and -1 for $\mathbf{g}_i^\ell$ determine the sign of the avoidance task. If $\mathbf{g}_i^\ell[i] = 1$, the avoidance task is negative and causes the joint to move away from its maximum limit $q_i^{\max}$. If $\mathbf{g}_i^\ell[i] = -1$, the avoidance task is positive and causes the joint to move away from its minimum limit $q_i^{\min}$. This can be explained by recalling that the projection operator has its diagonal with elements of non-negative values. Since the vector $\mathbf{g}_i^\ell$ for the joint j^i that nears its limits has only one non-zero element at its i^{th} component, then the sign of $(\mathbf{P}\mathbf{g}_i^\ell)[i]$ is the sign of the i^{th} component of $\mathbf{g}_i^\ell$ since both λ_{sec_i} and λ_{ℓ_i} have positive sign, as we will see in the following.

2) Tuning function: The tuning function $\lambda_{\ell_i}(q_i)$ depicted in Fig. 1 is used as a scaling parameter for the secondary task to control its injection onto the main task such that the final behavior of the robot system avoids any sudden movement due to the secondary task. For the joint under consideration j_i we define this tuning function based on two sigmoid functions as follows:

$$\lambda_{\ell i}(q_i) = \begin{cases} 1, & \text{if } \; q_i < q_{\ell_1 i}^{\min} \; \text{ or } \; q_{\ell_1 i}^{\max} < q_i \\ \frac{\lambda_{\ell i}^{\min}(q_i) - \lambda_{\ell_0 i}^{\min}}{\lambda_{\ell_1 i}^{\min} - \lambda_{\ell_0 i}^{\min}}, & \text{if } \; q_{\ell_1 i}^{\min} \leq q_i \leq q_{\ell_0 i}^{\min} \\ \frac{\lambda_{\ell i}^{\max}(q_i) - \lambda_{\ell_0 i}^{\max}}{\lambda_{\ell_1 i}^{\max} - \lambda_{\ell_0 i}^{\max}}, & \text{if } \; q_{\ell_0 i}^{\max} \leq q_i \leq q_{\ell_1 i}^{\max} \\ 0, & \text{if } \; q_{\ell_0 i}^{\min} < q_i < q_{\ell_0 i}^{\max} \end{cases} \tag{13}$$

where as previously defined, $q_{\ell_0 i}^{\min}$ and $q_{\ell_0 i}^{\max}$ are the threshold values to start the secondary task for avoidance. $q_{\ell_1 i}^{\min} = q_{\ell_0 i}^{\min} - \rho_1 \rho \Delta q_i$ and $q_{\ell_1 i}^{\max} = q_{\ell_0 i}^{\max} + \rho_1 \rho \Delta q_i$ are the threshold values at which the smoothing function equals one so that the avoiding task is totally injected into the main task, where $\rho_1 \in]0,1]$. $\lambda_{\ell i}^{\min}(q_i) : \mathbb{R} \rightarrow \mathbb{R}$ and $\lambda_{\ell i}^{\max}(q_i) : \mathbb{R} \rightarrow \mathbb{R}$ are two continuous monotonically increasing functions such that $\lambda_{\ell_1 i}^{\max} = \lambda_{\ell i}^{\max}(q_{\ell_1 i}^{\max}) \approx 1$, $\lambda_{\ell_0 i}^{\max} = \lambda_{\ell i}^{\max}(q_{\ell_0 i}^{\max}) \approx 0$, $\lambda_{\ell_1 i}^{\min} = \lambda_{\ell i}^{\min}(q_{\ell_1 i}^{\min}) \approx 1$ and $\lambda_{\ell_0 i}^{\min} = \lambda_{\ell i}^{\min}(q_{\ell_0 i}^{\min}) \approx 0$. A

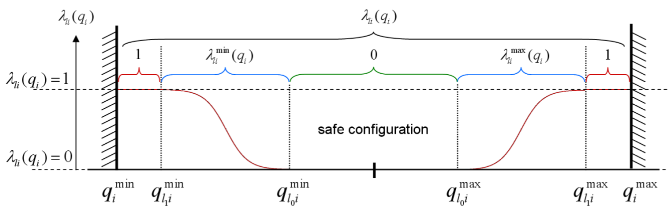


Fig. 1. Tunning function $\lambda_{\ell_i}(q_i)$ for the joint j_i

good selection for the functions $\lambda_{\ell i}^{\max}(q_i)$ and $\lambda_{\ell i}^{\min}(q_i)$ can be obtained using a sigmoid function as:

$$\lambda_{\ell i}^{\max}(q_i) = \frac{1}{1 + \exp(-12 \frac{q_i - q_{\ell_0 i}^{\max}}{q_{\ell_1 i}^{\min} - q_{\ell_0 i}^{\max}} + 6)} \tag{14}$$

$$\lambda_{\ell i}^{\min}(q_i) = \frac{1}{1 + \exp(-12 \frac{q_i - q_{\ell_0 i}^{\min}}{q_{\ell_1 i}^{\min} - q_{\ell_0 i}^{\min}} + 6)} \tag{15}$$

Introducing $q_{\ell_1 i}^{\min}$ and $q_{\ell_1 i}^{\max}$ gives the following advantage. By selecting these values near from $q_i^{\min}$ and $q_i^{\max}$ respectively, the joint will never reach the joint limit so that there is no need to predict the next joint position in order to avoid passing the joint limit.

3) Adaptive gain function: The adaptive gain function λ_{sec_i} is used to adapt the magnitude of $|\dot{\mathbf{q}}_{\mathbf{2}}^i[i]|$ in order to compensate the corresponding component of the main task near a joint limit. For the joint j_i, λ_{sec_i} is defined from the current values of the i^{th} components of both $\dot{\mathbf{q}}_1$ and $(\mathbf{P}\mathbf{g}_i^\ell)$. It is defined by:

$$\lambda_{sec_i} = \begin{cases} (1+\lambda_i)\frac{|\dot{\mathbf{q}}_1[i]|}{|(\mathbf{P}\mathbf{g}_i^\ell)[i]|}, & \text{if } \ \mathbf{g}_i^\ell[i] \neq 0 \\ 0, & \text{else} \end{cases} \tag{16}$$

where $\lambda_i \geq 0$.

An advantage of using this gain function is that the value of $\dot{\mathbf{q}}_{\mathbf{2}}^i$ given by (11) is a function of $\dot{\mathbf{q}}_1$, so that at any time step, the value of $\dot{\mathbf{q}}_{\mathbf{2}}^i$ is compatible with $\dot{\mathbf{q}}_1$ to be sent to the joint j_i. It can be demonstrated that using the adaptive gain function λ_{sec_i} allows us to ensure that the magnitude of the i^{th} velocity component of the main task and the avoidance tasks satisfy the inequality $|\dot{\mathbf{q}}_{\mathbf{2}}^i[i]| > |\dot{\mathbf{q}}_1[i]|$ before reaching the limits $q_i^{\max}$ or $q_i^{\min}$ of the joint j_i. Indeed, since near a joint limit $\lambda_{\ell i}(q_i) = 1$ and $\lambda_{sec_i} = (1+\lambda_i)\frac{|\dot{\mathbf{q}}_1[i]|}{|(\mathbf{P}\mathbf{g}_i^\ell)[i]|}$ then by recalling (11) we obtain $|\dot{\mathbf{q}}_{\mathbf{2}}^i[i]| = |-(1+\lambda_i)\dot{\mathbf{q}}_1[i]| > |\dot{\mathbf{q}}_1[i]|$ where $\lambda_i > 0$. Therefore, the gain function λ_{sec_i} allows us to ensure that the joint limit will not be reached, this property being ensured without any gain tuning step.

4) Behavior analysis of $\dot{\mathbf{q}}_{\mathbf{2}}^i$: Now, we study more precisely the behavior of $\dot{\mathbf{q}}_{\mathbf{2}}^i$ for avoiding the limits of the joint j_i. By injecting (16) and (13) in (11) , we get:

$$\dot{\mathbf{q}}_{\mathbf{2}}^i = \begin{cases} -(1+\lambda_i)\frac{|\dot{\mathbf{q}}_1[i]|}{|(\mathbf{P}\mathbf{g}_i^\ell)[i]|}\mathbf{P}\mathbf{g}_i^\ell, & \text{if } C_1 \\ -\lambda_{\ell i}(q_i)(1+\lambda_i)\frac{|\dot{\mathbf{q}}_1[i]|}{|(\mathbf{P}\mathbf{g}_i^\ell)[i]|}\mathbf{P}\mathbf{g}_i^\ell, & \text{if } C_2 \\ 0, & \text{if } C_3 \end{cases} \tag{17}$$

where

$$\begin{aligned} C_1 &\equiv (q_i < q_{\ell_1 i}^{\min} \ \text{ or } \ q_{\ell_1 i}^{\max} < q_i) \\ C_2 &\equiv (q_{\ell_1 i}^{\min} \leq q_i \leq q_{\ell_0 i}^{\min} \ \text{ or } \ q_{\ell_0 i}^{\max} \leq q_i \leq q_{\ell_1 i}^{\max}) \\ C_3 &\equiv (q_{\ell_0 i}^{\min} < q_i < q_{\ell_0 i}^{\max}) \end{aligned} \tag{18}$$

By investigating (17), we can study the behavior of the i^{th} component of the secondary task $|\dot{\mathbf{q}}_{\mathbf{2}}^i[i]|$, (see Fig. 2). Within the two intervals $[q_{\ell_0 i}^{\max}, q_{\ell_1 i}^{\max}]$ and $[q_{\ell_1 i}^{\min}, q_{\ell_0 i}^{\min}]$, the value of $|\dot{\mathbf{q}}_{\mathbf{2}}^i[i]|$ is changing continuously from 0 to $|(1+\lambda_i)\dot{\mathbf{q}}_1[i]|$ as the value q_i of the joint j_i is changing continuously from $q_{\ell_0 i}^{\max}$ to $q_{\ell_1 i}^{\max}$ and from $q_{\ell_0 i}^{\min}$ to $q_{\ell_1 i}^{\min}$ respectively. Within the two intervals $[q_{\ell_1 i}^{\max}, q_i^{\max}]$ and $[q_i^{\min}, q_{\ell_1 i}^{\min}]$, the value of $|\dot{\mathbf{q}}_{\mathbf{2}}^i[i]|$ is greater than or equal to $|\dot{\mathbf{q}}_1[i]|$. The value of the constant λ_i is used to tune the difference in the magnitude between $|\dot{\mathbf{q}}_1[i]|$ and $|\dot{\mathbf{q}}_{\mathbf{2}}^i[i]|$. For example, if $\lambda_i = 0$, we get $|\dot{\mathbf{q}}_{\mathbf{2}}^i[i]| = |\dot{\mathbf{q}}_1[i]|$ as soon as the value of the joint j_i reaches $q_{\ell_1 i}^{\max}$ or $q_{\ell_1 i}^{\min}$. While if $\lambda_i = 1$, as soon as the value of q_i approaches $q_{\ell_1 i}^{\max}$ or $q_{\ell_1 i}^{\min}$ we obtain $|\dot{\mathbf{q}}_{\mathbf{2}}^i[i]| = 2|\dot{\mathbf{q}}_1[i]|$. Fig. 2 illustrates the relations between i^{th} velocity component of $\dot{\mathbf{q}}_2$ and the i^{th} velocity component of $\dot{\mathbf{q}}_1$ for all values of $q_i \in [q_i^{\min}, q_i^{\max}]$ at the joint j_i. It is ensured by this investigation that, near to a limit of a robot joint j_i, $|\dot{\mathbf{q}}_{\mathbf{2}}^i[i]| \geq |\dot{\mathbf{q}}_1[\mathbf{i}]|$.

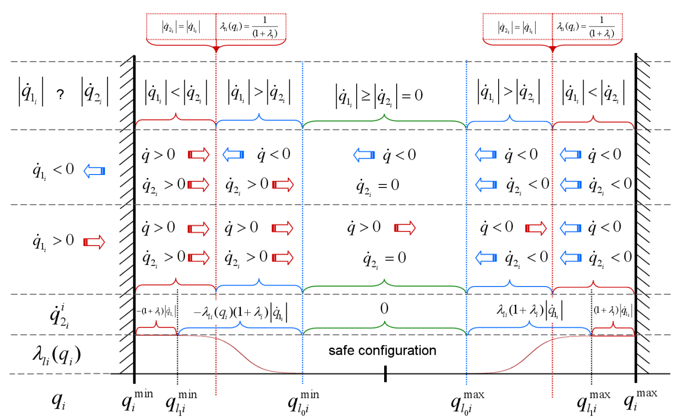


Fig. 2. Comparison between the magnitudes of the i^{th} components $\dot{\mathbf{q}}_2[i]$ and $\dot{\mathbf{q}}_1[i]$ where $\dot{\mathbf{q}}_{\mathbf{2}}^i[i]$ is defined by (17) within the different values for q_i of the joint j_i and when $\lambda_i > 0$; Illustration of the relation between the direction of $\dot{\mathbf{q}}_1[i]$, $\dot{\mathbf{q}}_{\mathbf{2}}^i[i]$, and $\dot{\mathbf{q}}[i]$.

IV. Control Scheme for Joint Limit Avoidance and Additional Secondary Tasks

Now, we consider the case when there are three tasks to be performed by the system. The three tasks consists of the main task; the secondary task for robot joint limits avoidance, and an additional secondary task. In this case, the control law is defined as:

$$\dot{\mathbf{q}} = \dot{\mathbf{q}}_1 + \dot{\mathbf{q}}_2 + \dot{\mathbf{q}}_3 \tag{19}$$

where $\dot{\mathbf{q}}_\mathbf{1}$ is the velocity vector due to the main task, $\dot{\mathbf{q}}_\mathbf{2}$ is the velocity vector due to the joint limits avoidance and finally $\dot{\mathbf{q}}_\mathbf{3}$ is the velocity vector due to the additional secondary task. More precisely, we get:

$$\dot{\mathbf{q}} = -\lambda_1 \mathbf{J_e}^+ \mathbf{e} - \sum_{i=1}^{n} \lambda^a_{sec_i} \lambda_{\ell_i} \mathbf{P} \mathbf{g}^\ell_i - \lambda_3 \mathbf{P} \mathbf{g_3} \tag{20}$$

Since the additional secondary task $\dot{\mathbf{q}}_\mathbf{3} = -\lambda_3 \mathbf{P}\mathbf{g_3}$ can lead to reach a robot joint limit, it is necessary to change the definition of $\lambda^a_{sec_i}$ in (16) to consider $\dot{\mathbf{q}}_\mathbf{3}$. In that case $\lambda^a_{sec_i}$ is defined by:

$$\lambda^a_{sec_i} = \begin{cases} (1+\lambda_i)\frac{|\dot{\mathbf{q}}_\mathbf{1}[i]+\dot{\mathbf{q}}_\mathbf{3}[i]|}{|\mathbf{P}\mathbf{g}^\ell_i[i]|} & \text{if} \quad \mathbf{g}^\ell_i[i] > 0 \\ 0, & \text{else} \end{cases} \tag{21}$$

This form for the adaptive gain ensures that the total velocity component sent to the joint j_i by $\dot{\mathbf{q}}_\mathbf{1}$ and $\dot{\mathbf{q}}_\mathbf{3}$ are controlled by $\dot{\mathbf{q}}_\mathbf{2}$ for joint limit avoidance. The avoidance task in this case is given by:

$$\dot{\mathbf{q}}^i_\mathbf{2} = \begin{cases} -(1+\lambda_i)\frac{|\dot{\mathbf{q}}_1[i]+\dot{\mathbf{q}}_3[i]|}{|(\mathbf{P}\mathbf{g}^\ell_i)[i]|}\mathbf{P}\mathbf{g}^\ell_i, & \text{if } C_1 \\ -\lambda_{\ell i}(q_i)(1+\lambda_i)\frac{|\dot{\mathbf{q}}_1[i]|+\dot{\mathbf{q}}_3[i]|}{|(\mathbf{P}\mathbf{g}^\ell_i)[i]|}\mathbf{P}\mathbf{g}^\ell_i, & \text{if } C_2 \\ 0, & \text{if } C_3 \end{cases} \tag{22}$$

To analyze the behavior of the avoidance task $\dot{\mathbf{q}}^i_\mathbf{2}$ near a joint limit, it is sufficient to replace $\dot{\mathbf{q}}_1$ by $\dot{\mathbf{q}}_1 + \dot{\mathbf{q}}_3$ which will lead to the same behavior study as in III-B.4. The adaptive gain $\lambda^a_{sec_i}$ given by (21) can be modified in the same manner when more additional secondary tasks have to be considered. The avoidance task will be given by:

$$\dot{\mathbf{q}}^i_\mathbf{2} = \begin{cases} -(1+\lambda_i)\frac{|\dot{\mathbf{q}}_1[i]+\sum_{k=3}^{k_{max}}\dot{\mathbf{q}}_k[i]|}{|(\mathbf{P}\mathbf{g}^\ell_i)[i]|}\mathbf{P}\mathbf{g}^\ell_i, & \text{if } C_1 \\ -\lambda_{\ell i}(q_i)(1+\lambda_i)\frac{|\dot{\mathbf{q}}_1[i]+\sum_{k=3}^{k_{max}}\dot{\mathbf{q}}_k[i]|}{|(\mathbf{P}\mathbf{g}^\ell_i)[i]|}\mathbf{P}\mathbf{g}^\ell_i, & \text{if } C_2 \\ 0, & \text{if } C_3 \end{cases} \tag{23}$$

Finally, the global control scheme is given by:

$$\dot{\mathbf{q}} = \dot{\mathbf{q}}_\mathbf{1} + \sum_{i=1}^{n} \dot{\mathbf{q}}^i_\mathbf{2} + \sum_{k=3}^{k_{max}} \dot{\mathbf{q}}_\mathbf{k} \tag{24}$$

where $\dot{\mathbf{q}}^i_\mathbf{2}$ is given by (23).

V. Experimental Results

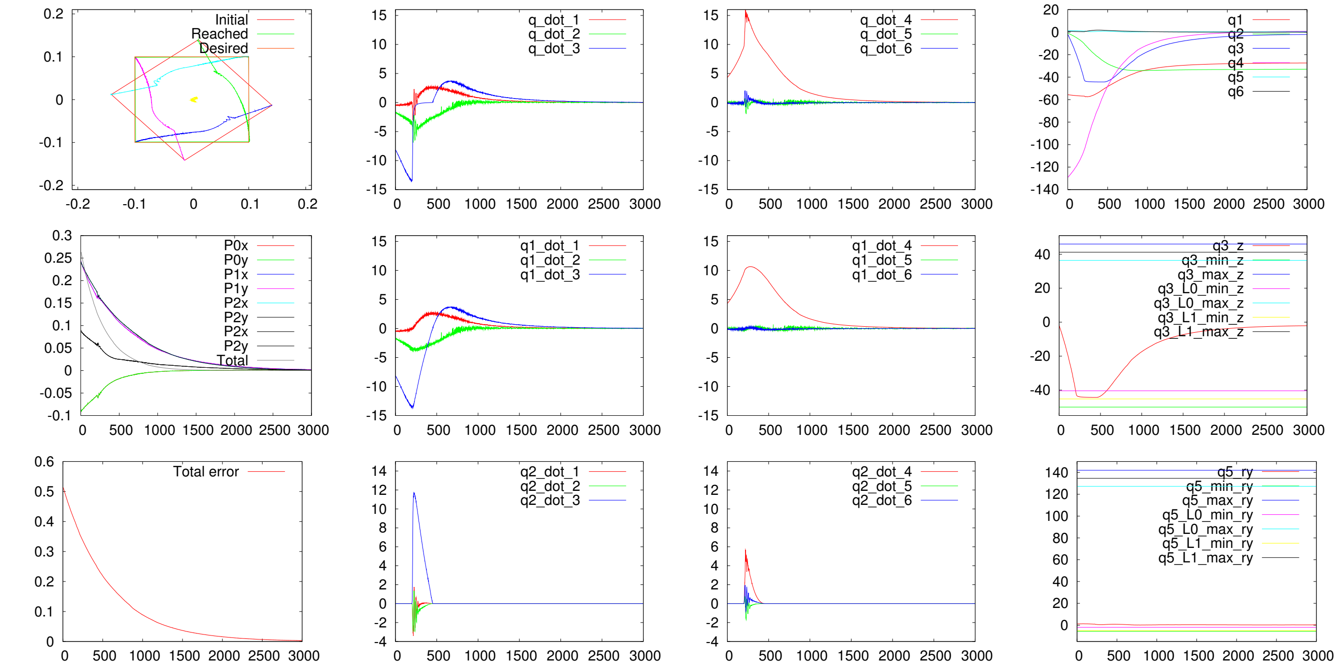
The experimental results presented in this section have been obtained using a Gantry robot having three prismatic joints and three revolute joints, ie. a six degrees of freedom robot arm. The values of the lower and upper limits for each joint are $q_1^{\min} = -0.7$, $q_1^{\max} = 0.7$, $q_2^{\min} = -0.6$, $q_2^{\max} = 0.63$, $q_3^{\min} = -0.5$, $q_3^{\max} = 0.46$, $q_4^{\min} = -156.41$, $q_4^{\max} = 156.0$, $q_5^{\min} = -5.73$, $q_5^{\max} = 142.0$, $q_6^{\min} = -91.0$, and $q_6^{\max} = 91.0$, where q_1, q_2, and q_3 are in meter and q_4, q_5, and q_6 are in degrees.

The proposed avoidance control scheme has been applied for a visual servoing task. In visual servoing [13], the task function is defined by $\mathbf{e} = \mathbf{s} - \mathbf{s}^*$ where $\mathbf{s}$ and $\mathbf{s}^* \in \mathbb{R}^m$ are two vectors representing the current and the desired selected visual features. The task Jacobian is $\mathbf{J_e} = \mathbf{L_s}\mathbf{M}\mathbf{J_q}$ where $\mathbf{L_s}$ is the interaction matrix that relates $\dot{\mathbf{s}}$ to the instantaneous camera velocity $\mathbf{v}$ by $\dot{\mathbf{s}} = \mathbf{L_s}\mathbf{v}$, $\mathbf{J_q}$ is the robot Jacobian and $\mathbf{M}$ is the matrix that relates $\mathbf{v}$ to the variation of the camera pose $\mathbf{p}$ by $\mathbf{v} = \mathbf{M}\dot{\mathbf{p}}$. The chosen task consists of positioning the camera with respect to a target composed of four points forming a square of length 0.1 m. The initial pose between the camera and the object has been chosen such that the camera is at a distance of 0.5 m from the object and the optical axis of the camera passes vertically through the center of the square such that the initial camera pose is (0,0,0.5,0,0,130). The desired camera pose is (0,0,0.5,0,0,0), which corresponds to a pure rotation of 130 degrees around the camera optical axis. These initial and desired configurations have been selected such that the main task causes the robot to perform a backward motion when image points coordinates are selected as visual features. By setting the initial configuration of the robot system such that the optical axis of the camera is parallel to the joint j_3 of the robot, a limit of the joint j_3 is reached.

All limit thresholds $q^{\min}_{\ell_0 i}$ and $q^{\min}_{\ell_1 i}$ are computed using the parameters $\rho = 0.1$ and $\rho_1 = 0.5$ as explained in Section III except for the lower limit parameters for the 5^{th} joint. Indeed, since the articular position for the initial configuration of the robot for our experiment is $\mathbf{q} = (-0.51, 0.09, -0.02, -128.89, 1.46, 0.01)$, we set $q^{\min}_{\ell_0 5} = -2$ and $q^{\min}_{\ell_1 5} = -5$ so that j_5 is not in a critical situation at the beginning of the experiments. The avoidance and secondary tasks are projected using the new projection operator $\mathbf{P}_{\|\mathbf{e}\|}$. Let us recall that it is impossible to use the classical projection operator $\mathbf{P_e}$ in all cases presented, since it is of rank 0. Furthermore, in all experiments, we never encountered the situation where $\mathbf{e} \in \text{Ker}(\mathbf{J_e}^\top)$, in which case $\mathbf{P}_{\|\mathbf{e}\|}$ is singular. Finally, we recall that we use a switching from $\mathbf{P}_{\|\mathbf{e}\|}$ to $\mathbf{P_e}$ when $\mathbf{e} \rightarrow \mathbf{0}$, as proposed in [14].

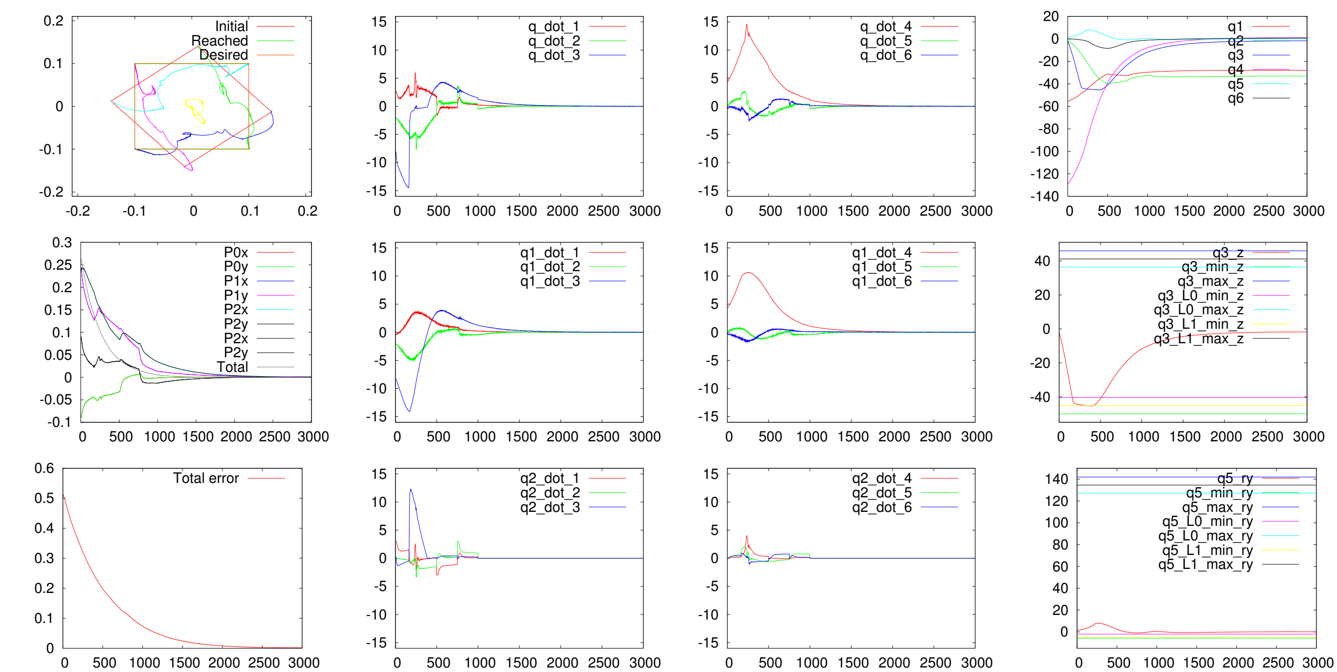
A. Case 1: Joint limits avoidance

In this case, the system starts its movement as expected from control (4) by a backward motion. When the threshold value $q^{\min}_{\ell_0 3}$ is reached, the avoidance is activated for the joint j_3 and the robot avoids reaching $q_3^{\min}$. As depicted in Fig. 3, $\dot{\mathbf{q}}_\mathbf{2}$ starts to have a non-zero value in the opposite direction of the main task within the iteration interval (250,420). During this interval, the velocity component of the global task $\dot{\mathbf{q}}$ for the joint j_3 is approximately equal to zero because of the effect of $\dot{\mathbf{q}}_\mathbf{2}$, while the rotation around the revolute joint j_4, which corresponds to the camera optical axis, is performed by the main task. Then, the direction of $\dot{\mathbf{q}}_\mathbf{1}[3]$ of the main task is inverted near to iteration number 420, so that it causes the robot to move away from its joint limit, that is why $\dot{\mathbf{q}}_\mathbf{2}$ stops since it becomes useless, although the value of $q_3 \in [q^{\min}_{\ell_0 3}, q_3^{\min}]$ within the iteration interval (420,500). This can be seen in Fig. 3 by noticing the values of the third component of $\dot{\mathbf{q}}_\mathbf{2}$ and q_3. Finally, the system converges to its desired position while ensuring the exponential decreasing for $\|\mathbf{e}\|$ all along the task, as expected [14].



(a) (b) (c) (d)

Fig. 3. Results for case 1. Pure avoidance, no additional secondary task. From up to down (a): the image points trajectories, image point error and the total error; ((b) and (c)): the articular velocity components of $\dot{\mathbf{q}}$, $\dot{\mathbf{q}}_1$, and $\dot{\mathbf{q}}_2$ respectively in cm/s and deg/s; (d): shows the values of the robot joints.

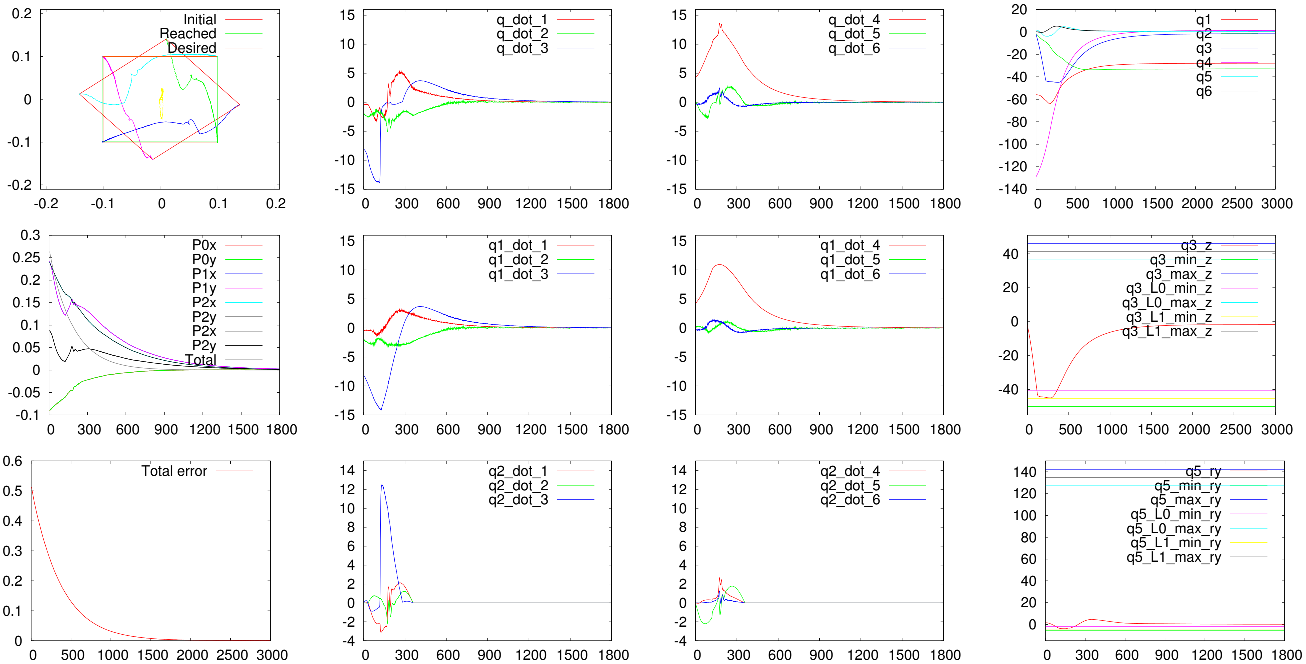


(a) (b) (c) (d)

Fig. 4. Results for case 2: Avoidance and additional secondary task sent to translational joints; q2_dot = $\dot{\mathbf{q}}_2 + \dot{\mathbf{q}}_3$

B. Case 2: Additional secondary task sent to prismatic joints

In this case, a motion that tries to move the end effector in a square of length 0.03 in the articular frame is specified as additional secondary task. The vector $\mathbf{g_3}$ that produces this motion is defined as $\mathbf{g_3} = (0.03, 0, 0, 0, 0, 0)$ when $(1 < iter < 250)$, $\mathbf{g_3} = (0, -0.03, 0, 0, 0, 0)$ when $(250 < iter < 500)$, $\mathbf{g_3} = (-0.03, 0, 0, 0, 0, 0)$ when $(500 < iter < 750)$, $\mathbf{g_3} = (0, 0.03, 0, 0, 0, 0)$ when $(750 < iter < 1000)$, else $\mathbf{g_3} = (0, 0, 0, 0, 0, 0)$. It concerns only the two prismatic joints j_1 and j_2. As depicted in Fig. 4, q2_dot represents the avoidance task combined with the projection of $\mathbf{g_3}$. As expected, secondary motions along x-axis and y-axis are produced during the first 1000 iterations due to the projection of the vector $\mathbf{g_3}$. It is clear that even if the error of each feature lost its exponential decrease mainly due to the effect of $\dot{\mathbf{q}}_3$, the servo system keeps the exponential decrease for the norm of the total error, as expected thanks to $\mathbf{P}_{\|\mathbf{e}\|}$. Again, the visual servoing task is performed successfully by



(a) (b) (c) (d)

Fig. 5. Results for case 3, Avoidance and additional secondary task sent to a revolute joint; q2_dot = $\dot{\mathbf{q}}_\mathbf{2} + \dot{\mathbf{q}}_\mathbf{3}$

avoiding the joint limit of j_3.

C. *Case 3: Additional secondary task sent to a revolute joint*

In this case, a sinusoidal motion defined by $f(t) = \frac{-3\pi}{180}(\sin(\frac{t*\pi}{180}))$ on the revolute joint j_5 is chosen as additional secondary task. Through the first 360 iterations, the vector of this secondary task is given by $\mathbf{g_3} = (0,0,0,0,f(t),0)$ where t is the number of iterations. The obtained results depicted on Fig. 5 show that the system succeeds to avoid two joint limits simultaneously, a joint limit of j_3 at $q_{\ell_0 3}^{\min}$ due to the main task and a joint limit of j_5 at $q_{\ell_0 5}^{\min}$ due to the additional task. Again, as illustrated in Fig. 5, the system keeps the exponential decreasing of the norm of the total error while projecting the additional secondary task $\mathbf{g_3}$ and avoiding the joint limits thanks to the new projection operator $\mathbf{P}_{\|\mathbf{e}\|}$.

VI. Conclusions

A new avoidance scheme for avoiding joint limits has been proposed in this paper. It is based on three functions: an activation function that activates the avoidance task and sets the direction of its actions, an adaptive gain function that controls the magnitude of the avoidance task, and a tuning function that ensures the smoothness of the injection of the avoidance task into the main task. The avoidance strategy has been considered with the new large projection operator proposed in [14]. Considering several additional secondary tasks has also been proposed.

The avoidance method proposed in this paper has been implemented and validated on a six DOFs robot arm. A nice behavior has been obtained that makes the system avoid the joint limits very smoothly even when the main task constrains all the robot degrees of freedom and when additional secondary tasks are considered.

References

[1] H Zghal, R V Dubey, and J A Euler, "Efficient gradient projechon optimization lor manipulators with multiple degrees of redundancy", *Proc IEEE Int. Conf Robotics and Automation* Vol(2):1006-1011, 1990.

[2] J. Lenarcic, "Effective secondary task execution of redundant manipulators", *Robotica* 16(4):457-462, 1998.

[3] O. Khatib, "The impact of redundancy on the dynamic performance of robots", *Laboratory Robotics and Automation*, 8:37-48, 1996.

[4] B. Siciliano and J-J. Slotine. "A general framework for managing multiple tasks in highly redundant robotic systems", *Proc IEEE Int. Conf Robotics and Automation* pp. 12111216, 1991.

[5] H. Hanafusa, T.Yoshikawa, and Y. Nakamura, "Analysis and control of articulated robot with redundancy", *Proc. IFAC, 8th Triennal World Congress*, vol. 4, pp. 1927-1932, Kyoto, Japan, 1981.

[6] A. Liegeois, "Automatic supervisory control of the configuration and behavior of multibody mechanisms", *IEEE Trans. on Systems, Man and Cybernetics*, 7(12):868-871, Dec. 1977.

[7] E. Marchand, F. Chaumette, A. Rizzo "Using the task function approach to avoid robot joint limits and kinematic singularities in visual servoing", *In IEEE/RSJ Int. Conf. on Intelligent Robots and Systems* , Vol(3), page(s): 1083-1090, November 1996.

[8] T. F. Chan and R. V. Dubey "A Weighted least-norm solution based scheme for avoiding joint limits for redundant joint manipulators", *IEEE Trans. on Automatic Control*, Vol(11),page(s):286-292, April 1995.

[9] N. Mansard and F. Chaumette, "Directionnal redundancy for robot control", *IEEE Trans. on Automatic Control*, 54(6):1179-1192, june 2009.

[10] T. Yoshikawa, "Basic optimization methods of redundant manipulators" *Laboratory Robotics and Automation*. 8:1, 49-60, 1996.

[11] F. Chaumette and E. Marchand, "A redundancy-based iterative scheme for avoiding joint limits: Application to visual servoing", *IEEE Trans. on Robotic and Automation.*, 17(5):719, 2001.

[12] B. Nelson and P. Khosla, "Strategies for increasing the tracking region of an eye-in-hand system by singularity and joint limits avoidance", *Int. Journal of Robotics Research*, 14(3):255-269, June 1995.

[13] F. Chaumette and S. Hutchinson, " Visual servo control, part i: Basic approaches", *IEEE Robotics and Automation Magazine*, 13(4):82–90, December 2006.

[14] M. Marey and F. Chaumette "A new large projection operator for the redundancy framework", *IEEE Int. Conf. on Robotics and Automation, ICRA'10* , pp.3727-3732, May 2010.